

# Robustness Analyze of Moments for Images against Rotation, Translation and Noise

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## Abstract

Moments have widespread application in image processing because of their properties on image rotation and translation. One of the shortcoming of moments on images is that they will not resist against noisy images. In this paper at first we will identify moments features on images and then investigate there robustness against rotation, translation and AWGN. And at the end we propose a preprocessing method for noisy images and will show that this preprocessing will robust moments against AWGN.

**Keywords:** Moment, AWGN, Mean, Mass, Gyration.



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## Introduction

Moments are scalar quantities used for hundreds of years to characterize a function and to capture its significant features. They have been widely used in statistics for description of the shape of a probability density function. From the mathematical point of view, moments are “projections” of a function onto a polynomial basis (similarly, Fourier transform is a projection onto a basis of harmonic functions). For the sake of clarity, we introduce some basic terms and definitions of moments. By an image function (or image) we understand any piecewise continuous real function  $f(x,y)$  of two variables defined on a compact support  $D \subset \mathbb{R} \times \mathbb{R}$  and having a finite nonzero integral. General moment  $M_{pq}^{(f)}$  of an image  $f(x,y)$ , where  $p$  and  $q$  are non-negative integers and  $r = p+q$  is called the order of the moment, is defined as:

$$M_{pq}^{(f)} = \iint_D p_{pq}(x,y)f(x,y)dx dy \quad (1)$$

where  $p_{00}(x,y), p_{01}(x,y), \dots, p_{kj}(x,y), \dots$  are polynomial basis functions defined on  $D$ . Depending on the polynomial basis used, we recognize various systems of moments [1, 2, 3, 4].

In the case of an image, the moments can be define as follows:

$$m_0 = \sum_{x,y} p(x,y)f(x,y)^0 \quad (1)$$

$$m_1 = \sum_{x,y} p(x,y)f(x,y)^1 \quad (2)$$

$$m_2 = \sum_{x,y} p(x,y)f(x,y)^2 \quad (3)$$

$$m_3 = \sum_{x,y} p(x,y)f(x,y)^3 \quad (4)$$

$$m_4 = \sum_{x,y} p(x,y)f(x,y)^4 \quad (5)$$

That  $m_0$  indicates the mass,  $m_1$  indicates the mean,  $m_2$  indicates the gyration,  $m_3$  indicates the skewness and  $m_5$  indicates the kurtosis.  $f(x,y)$  is the value of image in pixel  $(x,y)$  and  $p(x,y)$  is the existence probability of  $f(x,y)$  in image values [5].

## 2. Rotation and Translation Robustness Analyze

Now by the definition of moments on images we will obtain these moments for a rotated images. The original image and the rotated one by the degree of  $90^\circ$  is shown in figure 1.



**Figure 1.** (a) The original image (b) the rotated image by the degree of  $90^\circ$

The value of moments are calculated and shown in table 1 for the original image and for image by  $90^\circ$  degree of rotation.

**Table 1.** Comparison of moments of the original image and the rotated one by the degree of  $90^\circ$ .

	Original image	Rotated image
$m_0$	156.2136	156.2136
$m_1$	2.39E+04	2.39E+04
$m_2$	3.91E+06	3.91E+06
$m_3$	6.57E+08	6.57E+08
$m_4$	1.12E+11	1.12E+11

As it is shown in table 1 the moments for the original image and the rotated one are the same that shows the robustness of moments against rotation. If we take another angle for rotation like the image in figure 2, by eliminating zero pixels (black areas) that do not refer to the original image it is seen that, the moments are the same again.(table 2)



**Figure 2.** (a) The original image (b) the rotated image by the degree of 30°

**Table 2.** Comparison of moments of the original image and the rotated one by the degree of 30°

	Original image	Rotated image
$m_0$	156.2136	156.2136
$m_1$	2.3927e + 04	2.3927e + 04
$m_2$	3.9076e+06	3.9076e+06
$m_3$	6.5693e+08	6.5693e+08
$m_4$	1.1211e + 11	1.1211e + 11

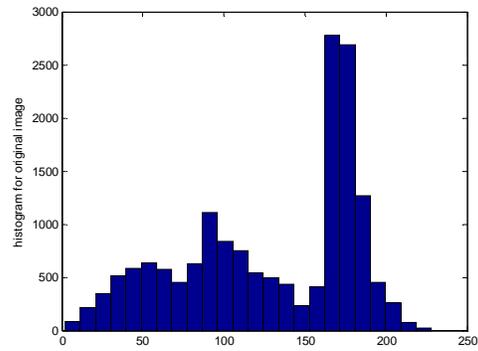
It is shown that if the original image is a part of a larger image, by translation of the original image, the moments of the translated image is the same as the original one. These results means that moments are robust against translation and rotation.

### 3. AWGN Robustness Analyze

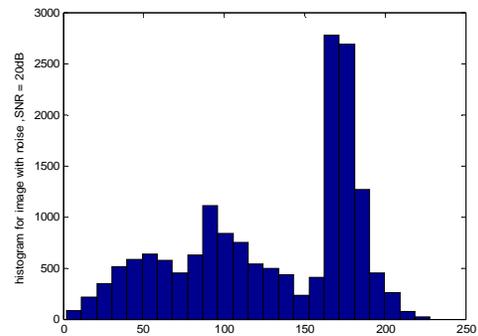
In this section at first we apply an Additive White Gaussian Noise (AWGN) with the SNR of 20 dB to the original image (Figure 3). The histogram of the original image and the noisy one is illustrated in figure 4 and 5.



**Figure 3.** Polluted image with AWGN



**Figure 4.** Histogram of original image



**Figure 5.** Histogram of the polluted image by AWGN of 20 dB

As it is seen from images when the SNR is high, the histograms are very similar to each other. The value of moments are shown in table 3.

**Table 3.** Comparison of moments of the original image and the polluted one by AWGN of 20 dB

	Original image	polluted image
$m_0$	156.2136	1
$m_1$	2.3927e + 04	128.2554
$m_2$	3.9076e+06	1.9283e+04
$m_3$	6.5693e+08	3.1258e+06
$m_4$	1.1211e + 11	5.2773e+08

Compare with the table 1 the values of moment have changed. To address this problem we apply the rounding pre-processing to noisy image after applying the noise and then calculate the moment's values that are seen in table 4.

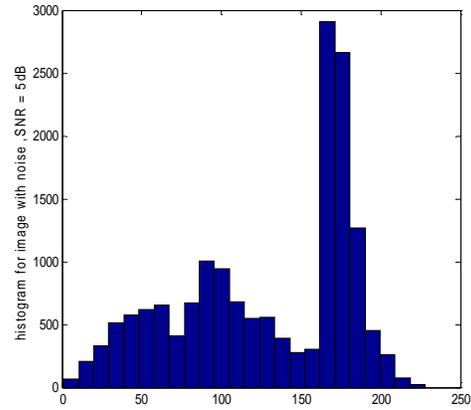
**Table 4:** Comparison of moments of the original image and the polluted one by AWGN of 20 dB with rounding preprocessing

	Original image	polluted image
$m_0$	156.2136	156.2136
$m_1$	2.3927e + 04	2.3927e + 04
$m_2$	3.9076e+06	3.9076e+06
$m_3$	6.5693e+08	6.5693e+08
$m_4$	1.1211e + 11	1.1211e + 11

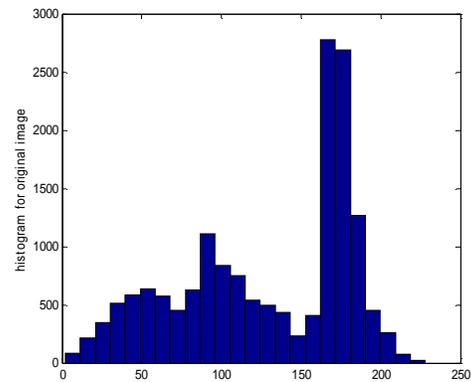
It is seen that with rounding preprocessing, the moment will be robust against AWGN. Now if we consider AWGN with 5dB SNR, the referred moments are shown in table 5 and its histogram is shown in figure 6. (To compare the difference between them, the histogram of the original image is also shown in figure 7). In comparison with the original image there is an obvious difference in their histograms.

**Table 5.** Comparison of moments of the original image and the polluted one by AWGN of 5 dB with rounding preprocessing

	Original image	polluted image
$m_0$	156.2136	154.7704
$m_1$	2.3927e + 04	2.3678e+04
$m_2$	3.9076e+06	3.8657e+06
$m_3$	6.5693e+08	6.4985e+08
$m_4$	1.1211e + 11	1.1090e+11



**Figure 6.** Histogram of the polluted image by AWGN of 3 dB



**Figure 7.** Histogram of original image

It is seen that in cases that the SNR is small by the method of rounding preprocessing although the values of moments will change but it is close to the values of the original image that shows the desire robustness of moments against AWGN.

## Conclusion

In this paper we investigate the robustness of moments for images against rotation, translation and AWGN. It is shown that the moments are very robust against translation and rotation. By adding AWGN to the image it is shown that the value of moments will change. To address this problem we use the rounding preprocessing for polluted images with 5 and 20 dB of AWGN noise and by means of it the moments got robust again.

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